

The Fundamental Identity and Reference Angles

The Fundamental Identity

The fundamental identity of trigonometry involves the squares of the sine and cosine function. We write $(\cos \alpha)^2$ as $\cos^2 \alpha$ and $(\sin \alpha)^2$ as $\sin^2 \alpha$. Remember that by definition, $\sin \alpha = y/r$, $\cos \alpha = x/r$, and $x^2 + y^2 = r^2$.

$$\sin^2 \alpha + \cos^2 \alpha = \frac{y^2}{r^2} + \frac{x^2}{r^2} = \frac{y^2 + x^2}{r^2} = \frac{r^2}{r^2} = 1$$

The Fundamental Identity of Trigonometry: If α is any angle or real number, then $\sin^2 \alpha + \cos^2 \alpha = 1$.

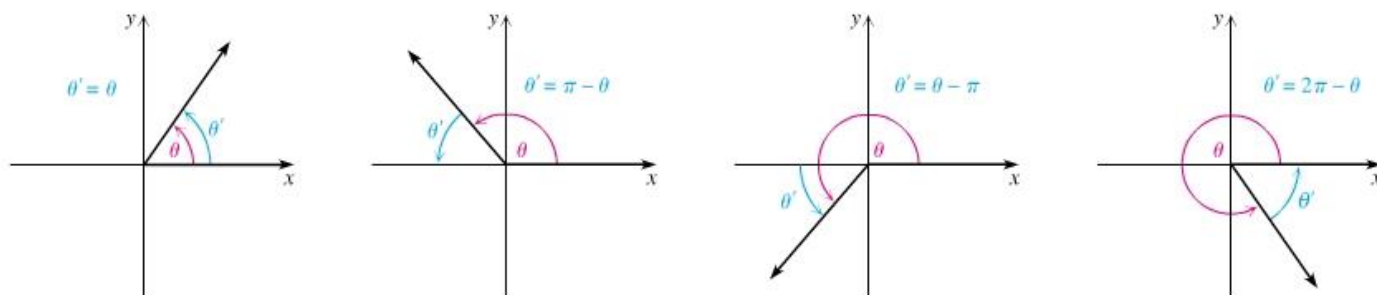
If we know the sine or cosine of an angle, then we can use the fundamental identity to find the value of the other function of the angle. (Note: you can also figure this out by drawing a triangle and using the Pythagorean Theorem).

Example: Find $\sin \alpha$ given that $\cos \alpha = 1/4$ and α is in Quadrant I.

Example: Find $\cos \alpha$ given that $\sin \alpha = -\sqrt{5}/3$ and α is in Quadrant III.

Reference Angles: When you look at the unit circle, notice that there is a pattern to the coordinates. If you look at all the angles that are 30° away from the x -axis (30° , 150° , 210° , 330°), the x -coordinate (cosine) is $\pm\sqrt{3}/2$ and the y -coordinate (sine) is $\pm 1/2$.

Definition: Reference Angle: If θ is a nonquadrantal angle (not on an axis) in standard position, then the reference angle θ' (read “theta prime”) formed by the terminal side of θ and the positive or negative x -axis.



Examples: For each given angle θ , sketch the reference angle θ' and give the measure of θ' in both radians and degrees.

$$\theta = 120^\circ$$

$$\theta = 7\pi/6$$

$$\theta = 690^\circ$$

$$\theta = -7\pi/4$$

Evaluating Trigonometric Functions Using Reference Angles: For an angle θ in standard position that is not a quadrantal angle:

$$\sin \theta = \pm \sin \theta', \quad \cos \theta = \pm \cos \theta', \quad \tan \theta = \pm \tan \theta',$$

$$\csc \theta = \pm \csc \theta', \quad \sec \theta = \pm \sec \theta', \quad \cot \theta = \pm \cot \theta'$$

where θ' is the reference angle for θ and the sign is determined by the quadrant in which θ lies.

Examples: Find the sine and cosine for each angle using reference angles.

$$\theta = 120^\circ$$

$$\theta = 7\pi/6$$

$$\theta = 690^\circ$$

$$\theta = -7\pi/4$$

Modeling with the Sine Function

The trigonometric functions can be used to model periodic phenomena.

Examples: Demand for a seasonal product can be modeled by the function $d = 200 \sin \frac{\pi(t-3)}{6} + 300$, where d is the number of units sold in month t . Find the demand in March ($t = 3$) and June ($t = 6$).